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Reply

Wind Energy Input to the Ekman-Stokes Layer: Reply to Comment by Jeff A. Polton

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This reply corrects the estimate of the wind and wave energy inputs into the Ekman layer by using the Ekman-Stokes layer energy budget as suggested in Polton (2009). Using the data and method in Liu *et al.* (2007), the global wind energy input was recalculated. The estimated global total energy input to the Ekman-Stokes layer is 2.22 TW, including 1.93 TW direct wind energy input and 0.29 TW wave-induced energy input. Compared to that in Liu *et al.* (2007), the recalculated wave-induced energy input was increased by 0.03 TW.

Keywords:
· Wind energy input,
· Ekman-Stokes layer,
· Coriolis-Stokes forcing.

1. Introduction

We recently published a paper on "Global estimates of wind energy input to subinertial motions in the Ekman-Stokes layer" (Liu *et al*., 2007; Hereinafter L07), investigating the implication of including a Coriolis-Stokes forcing in the energy equation for wave filtered Ekman flows. Similar approaches have also recently been used to estimate the wind and wave energy input into the Ekman layer within the Antarctic Circumpolar Current (Wu and Liu, 2008). We welcome the comment by Polton (2009; Hereinafter P09) addressing the wave-averaged energy equation, which is fundamental to our study. The direct wind and wave-induced energy inputs into the Ekman-Stokes layer have thus been re-estimated in this reply by using the Ekman-Stokes layer energy balance as suggested in P09.

In L07, the wave-affected energy equation is obtained by taking the inner product of the wave-filtered momentum equation (equation (2) in L07) with the Eulerian velocity. Although the formulation of the energy equation is mathematically correct, this procedure would lead to omission of the energy terms arising from correlation between the wave varying components of the same terms (P09). As P09 pointed out, it is essential that the energy equation should be obtained before taking the wave average. Thus, the wave-averaged energy balance equation for the Ekman-Stokes layer is (see also equations (20)–(24) in P09)

$$\frac{\partial E}{\partial t} = E_w + E_s - D, \tag{1}$$

where

$$E = \rho_w \int_{-\infty}^{0} \frac{1}{2} |\boldsymbol{U}|^2 dz, \tag{2}$$

$$E_w = \boldsymbol{\tau} \cdot \boldsymbol{U}(0), \tag{3}$$

$$E_s = -\rho_w \boldsymbol{f} \times \boldsymbol{T}_s(0) \cdot \boldsymbol{U}(0), \tag{4}$$

$$D = \rho_w \int_{-\infty}^{0} A_z \left| \frac{\partial \boldsymbol{U}}{\partial z} \right|^2 dz, \tag{5}$$

and

$$\boldsymbol{T}_s(z) = \int_{-\infty}^{z} \boldsymbol{U}_s(z) dz$$

is the Stokes transport. Here E represents the total kinetic energy of the Ekman-Stokes layer, E_w the rate of direct wind energy input, E_s the rate of energy input caused

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by the interaction of Stokes drift with planetary vorticity, and D the dissipation rate. All the above variable notations can be found in L07 or P09. It can be seen that the direct wind energy input term E_w and the dissipation term D are the same as those in L07. Thus, the decomposition of E_w into three terms and the global estimates of direct wind energy input in L07 are still valid under the new wave-averaged energy equation. The only difference in the wave filtered Ekman layer energy balance as between L07 and P09 is the wave-induced energy input term E_s, the physical meaning of which is the work done by the vertically averaged Coriolis-Stokes forcing on the sea surface Ekman flow. According to the following decomposition (also see equation (25) in P09)

$$\boldsymbol{f} \times \boldsymbol{T}_s(0) \cdot \boldsymbol{U}(0) = \int_{-\infty}^{0} \boldsymbol{f} \times \boldsymbol{T}_s \cdot \boldsymbol{U} dz + \int_{-\infty}^{0} \boldsymbol{f} \times \boldsymbol{T}_s \cdot \frac{\partial \boldsymbol{U}}{\partial z} dz, \quad (6)$$

the new wave-induced energy input term contains two parts: the first part is the same as that in L07, while the second one is missing in L07, where the energy balance is obtained directly from the wave-averaged momentum equation.

2. Energy Input to a Steady State Ekman-Stokes Layer

As in L07, substituting the steady solution of the Ekman-Stokes layer (Polton *et al.*, 2005; L07) into Eq. (3), the direct wind energy input E_w can thus be obtained, which is the same as equation (9) in L07. Similarly, the wave-induced energy input term can be obtained as (see also equation (A1) in P09 for the northern hemisphere only)

$$E_s = \rho_w |f| d_s |\boldsymbol{U}_s(0)|^2 F_2(c) + \frac{1}{c} \boldsymbol{\tau} \cdot \boldsymbol{U}_s(0) \pm \frac{1}{c} \hat{z} \cdot (\boldsymbol{\tau} \cdot \boldsymbol{U}_s(0)) \begin{pmatrix} +: f > 0 \\ -: f < 0 \end{pmatrix}. \quad (7)$$

The new wave-induced energy input term can also be partitioned into three terms:

$$E_{s,1} = \rho_w |f| d_s |\boldsymbol{U}_s(0)|^2 F_2(c),$$

$$E_{s,2} = \frac{1}{c} \boldsymbol{\tau} \cdot \boldsymbol{U}_s(0),$$

and

$$E_{s,3} = \pm \frac{1}{c} \hat{z} \cdot (\boldsymbol{\tau} \cdot \boldsymbol{U}_s(0)) \begin{pmatrix} +: f > 0 \\ -: f < 0 \end{pmatrix},$$

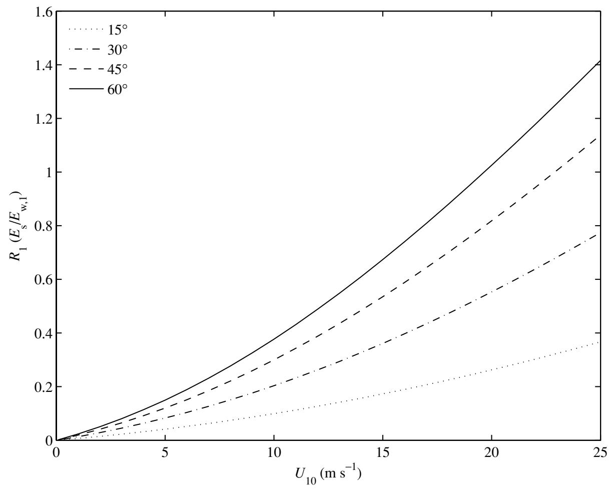

Fig. 1. Graph showing, respectively, the ratio of E_s to $E_{w,1}$ (R_1) varying with wind speed U_{10} from 1 to 25 m s^{-1} for four different latitudes: 15°, 30°, 45° and 60°.

where the expression for $F_2(x)$ is (same as in L07):

$$F_2(x) = \frac{x}{(x+1)^2 + 1}.$$

Obviously, the differences between the new and previous wave-induced energy input terms exist in the coefficients, which are different functions of the non-dimensional Ekman-Stokes depth number c. The total wind energy input into the Ekman-Stokes layer is

$$\begin{aligned} E_{tot} &= E_w + E_s \\ &= \frac{|\boldsymbol{\tau}|^2}{\rho_w d_e |f|} + \rho_w |f| d_s |\boldsymbol{U}_s(0)|^2 F_2(c) + \boldsymbol{\tau} \cdot \boldsymbol{U}_s(0) \left[\frac{1}{c} - F_1(c) \right] \\ &\pm \hat{z} \cdot (\boldsymbol{\tau} \times \boldsymbol{U}_s(0)) \left[\frac{1}{c} + F_2(c) \right] \begin{pmatrix} +: f > 0 \\ -: f < 0 \end{pmatrix}. \end{aligned} \quad (8)$$

To examine the relative importance of the wave contribution to the energy budget of the Ekman-Stokes layer, two ratios associated with the wave-induced energy input E_s are considered. Here we only consider the case that the wind direction is exactly the same as that of the waves. The first ratio R_1 of E_s to $E_{w,1}$ compares the wave-induced energy input with the wind energy input into the Ekman layer without wave effects included. The second ratio R_2 of E_s to E_w gives a measure of wave-induced energy input compared to the direct wind energy within the Ekman-Stokes layer. Like the approaches in

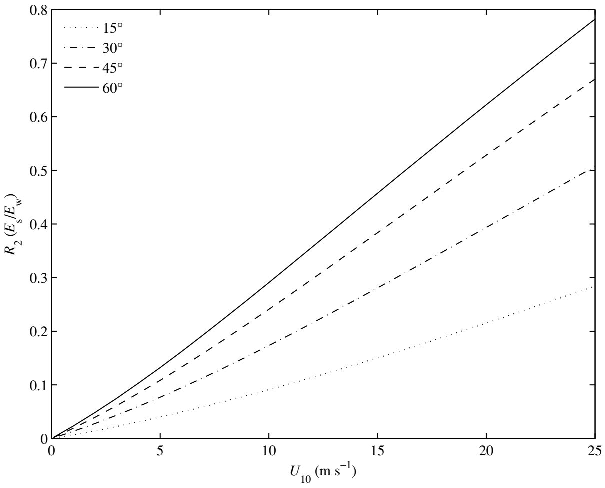

Fig. 2. Graph showing, respectively, the ratio of E_s to E_w (R_2) changing with wind speed U_{10} from 1 to 25 m s^{-1} for four different latitudes: 15°, 30°, 45° and 60°.

Wu and Liu (2008), the Ekman layer depth d_e and the Stokes drift $\boldsymbol{U}_s$ can be empirically estimated from the friction velocity in the air u_{*a}. The wind stress is parameterized through $|\boldsymbol{\tau}| = \rho_a C_D U_{10}{}^2 = \rho_a u_{*a}{}^2$, where U_{10} is the sea surface wind speed at 10-m height and C_D is the atmospheric drag coefficient which can be parameterized with U_{10}. Therefore, the ratios R_1 and R_2 can both be computed from U_{10}.

Figure 1 shows how R_1 increases with wind speed varying from 1 to 25 m s^{-1} for four different latitudes: 15°, 30°, 45° and 60°. It is indicated that the wave-induced energy input could not be neglected in discussions of energetics of the surface mixed layer. Figure 2 presents the ratio of wave-induced energy input to direct wind energy input into the Ekman-Stokes layer. The significance of wave-induced energy input can be found even for normal wind conditions. In high latitude areas with high wind conditions, it is shown that the work done by the Coriolis-Stokes forcing on the mean flows could be as important as that done by the wind stress.

3. Energy Input to a Non-steady Ekman-Stokes Layer

As for the wind energy input to a non-steady Ekman-Stokes layer, following the method in L07, the direct wind energy input for the n-th component is same as that in L07 (see their equation (14)), whereas the wave-induced energy input to a non-steady Ekman-Stokes layer for the n-th component can be expressed as

$$E_s^n = E_{s,1}^n + E_{s,2}^n + E_{s,3}^n, \tag{9}$$

where

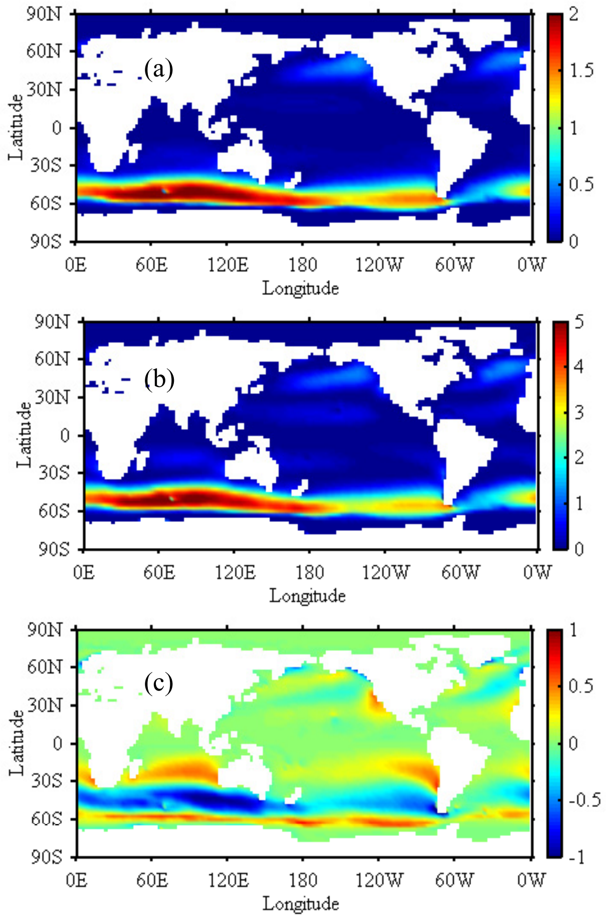

Fig. 3. Global distributions of (a) the first term $E_{s,1}$, (b) the second term $E_{s,2}$, and (c) the third term $E_{s,3}$ in the wave-induced energy input to the Ekman-Stokes layer (mW m^{-2}), with cutoff frequency at 0.5 cycle day^{-1}.

$$E_{s,1}^n = \rho_w \frac{f^2}{|f+\omega_n|} d_s^n |\boldsymbol{X}_n(0)|^2 F_2(c_n),$$

$$E_{s,2}^n = \frac{f}{f+\omega_n}\left(\boldsymbol{T}_n \cdot \boldsymbol{X}_n(0)\right)\frac{1}{c_n},$$

and

$$E_{s,3}^n = \pm \frac{f}{f+\omega_n}\hat{z}\cdot\left(\boldsymbol{T}_n \times \boldsymbol{X}_n(0)\right)\frac{1}{c_n}\begin{pmatrix} +: f+\omega_n > 0 \\ -: f+\omega_n < 0 \end{pmatrix}. \tag{10}$$

The total energy input to a non-steady Ekman-Stokes layer can be obtained by

$$E_{tot} = \sum_n \left(E_w^n + E_s^n\right). \tag{11}$$

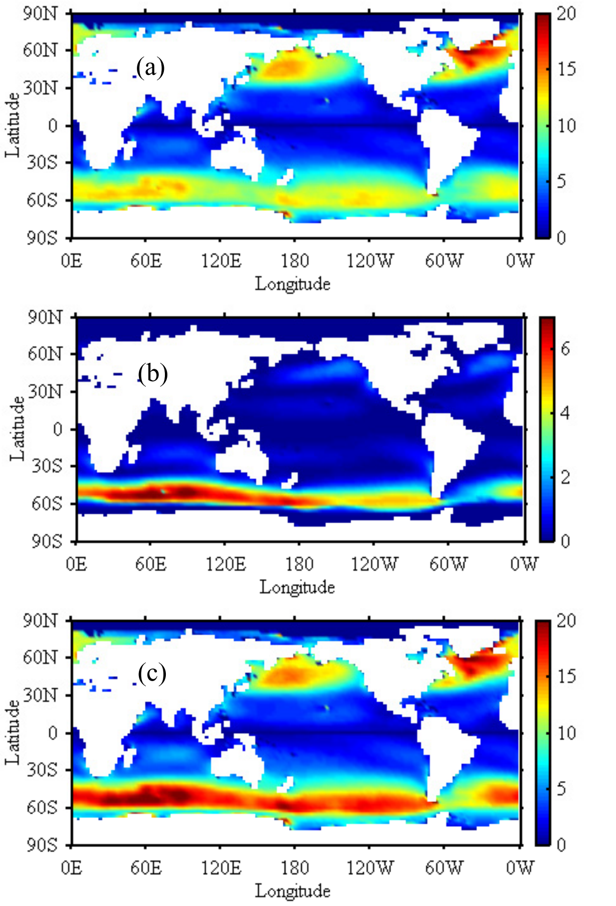

Fig. 4. Global distributions of (a) the direct wind energy input E_w, (b) the wave-induced energy input E_s, and (c) the total wind energy input E_{tot} to the Ekman-Stokes layer (mW m^{-2}), with cutoff frequency at 0.5 cycle day^{-1}.

Table 1. Distributions of the direct wind energy input E_w, the wave-induced energy input E_s, the total energy input E_{tot} to the Ekman-Stokes layer, with cutoff frequency at ω = 0.5 cycle day^{-1}. All energy inputs in TW.

		$\omega > 0$	$\omega = 0$	$\omega < 0$	Sum
E_w	Northern Hemisphere	0.26	0.09	0.38	0.73
	Southern Hemisphere	0.61	0.19	0.39	1.19
	Total	0.87	0.29	0.77	1.93
E_s	Northern Hemisphere	0.00	0.04	0.00	0.04
	Southern Hemisphere	0.00	0.25	0.00	0.25
	Total	0.00	0.29	0.00	0.29
E_{tot}	Northern Hemisphere	0.26	0.13	0.38	0.77
	Southern Hemisphere	0.61	0.45	0.39	1.45
	Total	0.87	0.58	0.77	2.22

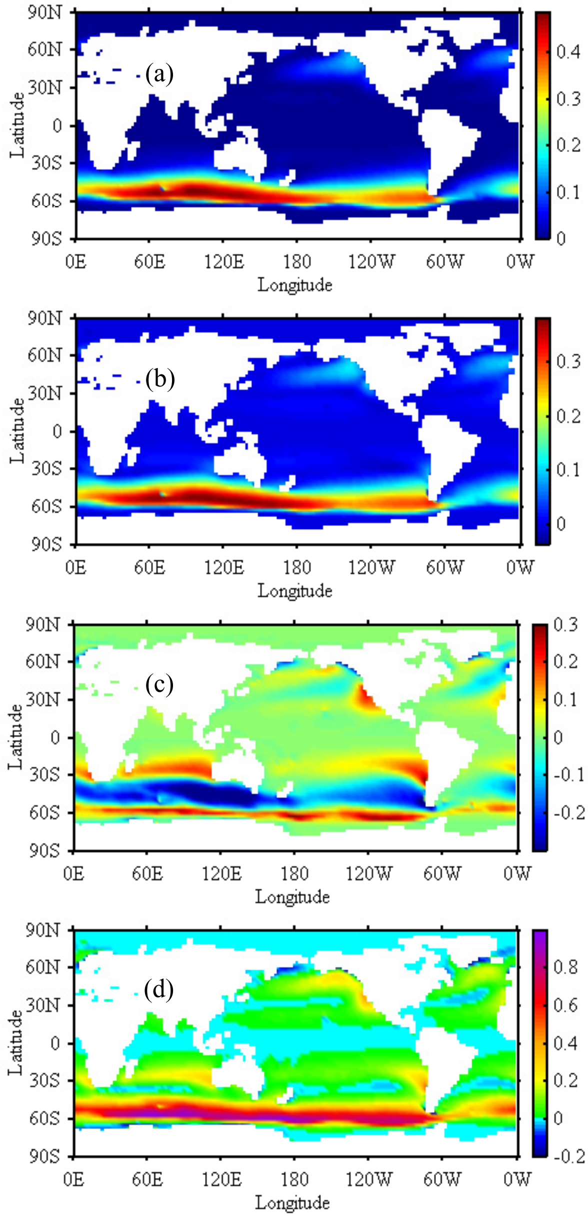

Fig. 5. The differences (mW m^{-2}) of (a) the first term $E_{s,1}$, (b) the second term $E_{s,2}$, (c) the third term $E_{s,3}$, and (d) the total E_s of the wave-induced energy input to the Ekman-Stokes layer, between the new and previous estimates. The cutoff frequency is 0.5 cycle day^{-1}.

4. Global Estimates of Wind Energy Input to the Ekman-Stokes Layer

The data and method used to estimate the wind energy input are the same as those given in L07. Since the direct wind energy input remains unchanged, we mainly focus on the wave-induced energy input in this reply. Figures 3(a)–(c) show the global distributions of the first, second and third terms ($E_{s,1}$, $E_{s,2}$, and $E_{s,3}$) in the wave-induced energy input to the Ekman-Stokes layer during 1958–2001, respectively. It can be seen that their distribution patterns are quite similar to those in L07 (Figs. 2(c) and (d)). The global distributions of the direct wind energy input E_w, the wave-induced energy input E_s, and the total energy input E_{tot} to the Ekman-Stokes layer are shown in Figs. 4(a)–(c), respectively. The direct wind energy input is the same as that in L07 (see their figure 1(a)). The distribution patterns of the wave-induced energy input and the total energy input are also similar to those in L07 (see their figures 1(b) and (c)). Details of the distributions of the direct wind energy input E_w, the wave-induced energy input E_s, and the total energy input E_{tot} to the Ekman-Stokes layer for the World Ocean are listed in Table 1. The direct wind energy input is 1.93 TW (same as that in L07). While considering the new energy balance, the wave-induced energy input is 0.29 TW, in contrast to 0.26 TW in L07. The global total wind energy input to the Ekman-Stokes layer thus reach 2.22 TW, 13% of it coming from the wave-induced energy input and 87% from the direct wind energy input.

As discussed in Sections 2 and 3, the differences between the new and previous wind energy inputs only exist in the coefficients of the three terms of wave-induced energy input. Figure 5 shows the distributions of the differences between the new and previous estimates for the first (a), second (b), third (c) terms and the total (d) of the wave-induced energy input. It is shown that the largest differences for the first and second terms of wave-induced energy input exist in the Antarctic Circumpolar Current (ACC) area (Figs. 5(a) and (b)). From Figs. 5(c) and 3(c) together with figure 2(d) in L07, it can be seen that in comparison with the estimation in L07 the newly-estimated third term of wave-induced energy input is increased at the locations where its value is positive, and is decreased at the locations where its value is negative. As a result, the largest differences in the total wave-induced energy input between the new and previous estimates are also located in the ACC area (Fig. 5(d)). Compared to that in L07, the newly-estimated total wave-induced energy input is increased by 0.03 TW.

5. Conclusions

This reply agrees that the Ekman-Stokes layer energy budget should be obtained first, prior to wave averaging, as pointed out in P09. The new wave-induced energy input to the Ekman-Stokes layer is then obtained. It is also decomposed into three terms, which are different from the previous ones (in L07) only in the coefficients. Using the data and method presented in L07, the global wind energy input was re-calculated. The estimated global total energy input to the Ekman-Stokes layer is 2.22 TW, including 1.93 TW direct wind energy input and 0.29 TW wave-induced energy input, which are slightly different from the figures published in L07. The recalculated wave-induced energy input is 0.03 TW larger than that given in L07.

Acknowledgements

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